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Squeezing and entangling atomic motion in cavity QED via quantum nondemolition measurement

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Abstract. We propose a scheme for preparing the squeezing of an atomic motion and an Einstein-Podolsky-Rosen state in position and momentum of a pair of distantly separated trapped atoms. The scheme utilizes the quantum nondemolition measurements with interaction between the cavity field and the motional state of the trapped atom in cavity QED. By illuminating the atoms with bichromatic light, the interaction Hamiltonian of the cross-Kerr effect between the cavity and atomic motion is generated to implement quantum nondemolition measurements.

PACS. 03.67.Hk Quantum communication – 32.80.Lg Mechanical effects of light on atoms, molecules, and ions – 42.50.-p Quantum optics

1 Introduction

The generation, distribution, and application of continuous variable quantum entanglement are topics of considerable interest at present in the fields of quantum communication and quantum computation. In this context, a variety of protocols for continuous quantum variables have been proposed and in some cases already demonstrated experimentally, including quantum teleportation [1], quantum cryptography [2], quantum dense coding [3], and quantum information [4]. The preparation of entangled atomic states is one of the goals of atomic physics and quantum optics. Various methods have been recently proposed to engineer entanglement between atoms [5,6].

The quantized motional states of atoms or ions in confining potentials offer interesting possibilities for a variety of applications, such as the preparation and study of nonclassical states [7] and the storage and manipulation of quantum information [8]. These possibilities step from the relatively long coherence times that can be achieved with motional states (due to the absence of strong damping mechanisms) and the precision with which transformations between motional states can be controlled using light-induced transitions. Recently, Parkins *et al.* proposed a scheme that enables motional quantum states to be coupled to propagating light fields *via* interactions in cavity quantum electrodynamics (cavity QED) [9]. The entanglement of the atoms' motional states is achieved through the transfer of entanglement from the quantum-correlated output light fields from a nondegenerate parametric amplifier [10]. To this end, more schemes were also proposed, one is that "local" entanglement of orthogonal motional modes of a single atom trapped inside an optical cavity is transformed *via* propagating light fields into "nonlocal" entanglement of the motional modes of distantly separated atoms [11], the other is that the motional modes of two trapped atom are entangled with a propagating light field *via* a cavity-mediated parametric interaction [12]. Mancini *et al.* exploited ponderomotive forces to entangle the motions of different atoms [13].

Quantum nondemolition (QND) measurement on samples of atoms has been proposed as a means to entangle atoms in the samples [14]. By QND detection atoms can be projected into an entangled state by the measurement and following these proposals experiments have produced and verified entanglement of large atomic samples [15]. In the present work, we propose a scheme to preparing the squeezed and entangled motional state by means of QND measurement. Two auxiliary lasers, incident through the sides of the cavities, combine with the cavity fields to drive Raman transitions between neighboring vibrational levels of the motion of each atom. The cross-Kerr effective interaction is generated for the QND measurement. In this way, the need of nondegenerate parametric amplifiers is eliminated, and all of the desired operations are achieved using only trapped-atom cavity QED configurations. The advantage of our scheme is that interaction quadratures of the cavity field and motional mode may be controllable by the bichromatic auxiliary lasers. So, the entangled states of two atomic motion are a bipartite entangled state with equal strength correlations between quadratures. Moreover, the controlled cross-Kerr interaction may be used directly for achieving some protocols for quantum communication between atomic motions.

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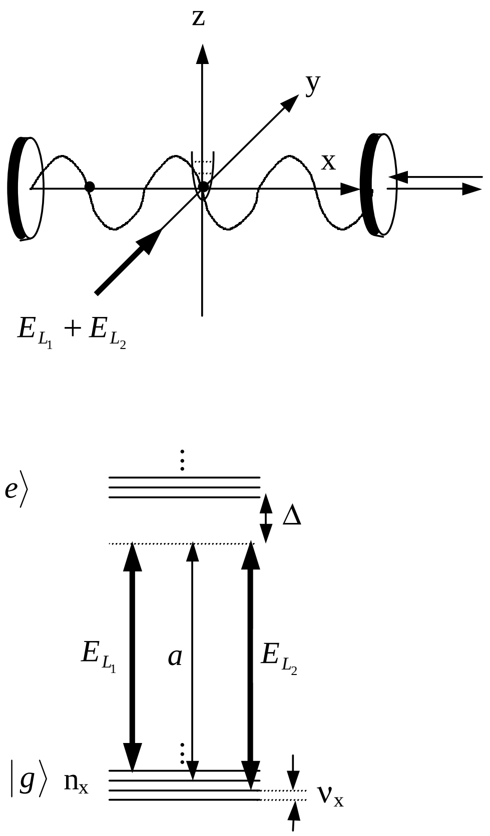

Fig. 1. Schematic of proposed experimental setup and excitation scheme.

2 Model

The basic setup we use was originally considered by Zeng and Lin [16] and developed for coupling motion to light by Parkins and Kimble [9,10]. This setup consists of a two-level atom confined in a harmonic trap located inside an optical cavity. The atomic transition of frequency ω_A is coupled to a single mode of the cavity field of frequency ω_c and is also driven by two external (classical) laser field of frequency ω_{L_1} and ω_{L_2}. The physical setup and excitation scheme are depicted in Figure 1. The cavity is aligned along the x-axis, while the laser field is incident from a direction along the y-axis (*i.e.*, perpendicular to the x-axis). The Hamiltonian describing the atom-cavity system, including the atomic motion, takes the form (in the frame rotating at the cavity field of frequency ω_c)

$$H = \hbar \sum_{i=x,y,z} v_i b_i^\dagger b_i + \hbar \varDelta \sigma_+ \sigma_- + \hbar \left[E_{\text{ext}} \sigma_+ + E_{\text{ext}}^* \sigma_- \right] + \hbar g_0 \sin(\kappa x) \left(a^\dagger \sigma_- + \sigma_+ a \right). \quad (1)$$

Here, $E_{\text{ext}} = E_{L_1} \mathrm{e}^{-(\mathrm{i}\delta_{L_1} t - \kappa y + \phi_{L_1})} + E_{L_2} \mathrm{e}^{-(\mathrm{i}\delta_{L_2} t - \kappa y + \phi_{L_2})}$ and E_{L_1}, E_{L_2} are the amplitude of laser fields. v_x, v_y, and v_z are the harmonic oscillation frequencies along the principal axis of the trap, b_i and a are annihilation operators for the quantized atomic motion and cavity field, respectively, $\sigma_- = |g\rangle \langle e|$ is the atomic lowering operator and $\varDelta = \omega_A - \omega_c$, $\delta_{L_1} = \omega_{L_1} - \omega_c$ and $\delta_{L_2} = \omega_{L_2} - \omega_c$. The single-photon atom-cavity dipole coupling strength is given by g_0 and κ is the wave number of the cavity field. The choice of sine function, with $x = (\hbar/2mv_x)^{1/2}(b_x + b_x^\dagger)$, being the position operator of the atom, denotes that the trap is assumed to be centered at a node of the cavity standing-wave field.

Heisenberg equations of motion are straightforwardly derived from the above Hamiltonian. Assuming the detunings of the light fields from the atomic transition frequency to be very large [*i.e.*, $\varDelta \gg v_j, g_0, |E_{L_1}|, |E_{L_2}|, |\delta_{L_1}|, |\delta_{L_2}|$], atomic spontaneous emission can be neglected and the internal atomic dynamics can be adiabatically eliminated. In the equations of motion, this is done by making the replacement

$$\sigma_- \simeq -\frac{1}{\varDelta}[E_{\text{ext}} + g_0 \sin(\kappa x) a]. \quad (2)$$

The corresponding Hamiltonian them takes the form

$$H_a = \hbar \sum_{i=x,y,z} v_i b_i^\dagger b_i + \frac{\hbar \left| E_{\text{ext}} \right|^2}{\varDelta} - \frac{\hbar g_0^2}{\varDelta} \sin^2(\kappa x) a^\dagger a - \frac{\hbar g_0}{\varDelta} \sin(kx) \left[E_{\text{ext}} a^\dagger + E_{\text{ext}}^* a \right]. \quad (3)$$

The second term shows that the bichromatic external fields do not drive the y motional sideband due to independence on y motion. The forth term describes the coupling between the cavity field mode and the bichromatic external fields. However, the bichromatic external fields can be not coupled directly to drive the motional sideband $2v_x$.

The size of the harmonic trap is assumed to be small compared to the optical wavelength (Lamb-Dicke approximation). This enables the approximations $\sin(\kappa x) \simeq \eta_x(b_x + b_x^\dagger)$. Given this assumption, it is also possible to design a configuration for which we can neglect all position dependence in the laser field. Therefore, the problem essentially becomes one-dimensional and we can restrict our attention to just the x-direction.

To second order in η_x, equations of motion for the operators a and b_x are then

$$\begin{aligned} \dot{a} &= -ka + \mathrm{i}\frac{\eta_x^2 g_0^2}{\varDelta} \left(b_x + b_x^\dagger \right)^2 a + \mathrm{i}\frac{g_0 \eta_x E_{\text{ext}}}{\varDelta} \left(b_x + b_x^\dagger \right) + \sqrt{2k} a_{\text{in}} \\ \dot{b}_x &= -\mathrm{i} v_x b_x + \mathrm{i}\frac{\eta_x^2 g_0^2}{\varDelta} \left(b_x + b_x^\dagger \right) a^\dagger a - \varGamma b_x + \zeta_x + \mathrm{i}\frac{g_0 \eta_x}{\varDelta} \left[E_{\text{ext}} a^\dagger + E_{\text{ext}}^* a \right]. \end{aligned} \quad (4)$$

Here, the operator a_{in} obeys the commutation relation $[a_{\text{in}}(t), a_{\text{in}}^\dagger(t')] = \delta(t - t')$ and describes the quantum noise input to the cavity field from the external field. The parameter k is the cavity field decay rate which occurs through the input-output mirror (we neglect other losses of cavity, such as losses from absorption, scattering of mirrors). The parameter $\varGamma$ is atomic motion decay rate. ζ_x describes the vacuum noise introduced by the atomic motion decay. We have to remark, however, that the damping and heating mechanisms of a trapped atom are not yet well understood [17] and that different kinds of ion-reservoir interaction have been proposed [18].

Next, we make the transformation $b_x = \widetilde{b}_x \mathrm{e}^{-\mathrm{i}v_x t}$, and choose $E_{L_1} = E_{L_2} = E_L$ and the detuning between the cavity and laser fields to be $-\delta_{L_1} = \delta_{L_2} = v_x$. The terms of second order in η_x describe photon-number dependent phase shift; these will in general be very small and may be neglected. Assuming that $v_x \gg k, |E_L g_0 \eta_x/\Delta|$, the oscillating terms in the resulting equations may be dropped in a rotating-wave approximation to yield

$$\begin{aligned}\dot{a} &= -ka + \mathrm{i}\Omega\left(\mathrm{e}^{-\mathrm{i}\phi_{L_1}}\widetilde{b}_x + \mathrm{e}^{-\mathrm{i}\phi_{L_2}}\widetilde{b}_x^{\dagger}\right) + \sqrt{2k}a_{\mathrm{in}} \\ \dot{\widetilde{b}}_x &= -\Gamma\widetilde{b}_x + \mathrm{i}\Omega\left(a^{\dagger}\mathrm{e}^{-\mathrm{i}\phi_{L_2}} + a\mathrm{e}^{\mathrm{i}\phi_{L_1}}\right) + \zeta_x \end{aligned} \tag{5}$$

where we have defined $\Omega = (g_0\eta_x E_L)/\Delta$. The quadratures of the cavity field and motional mode corresponding to equation (5) are then

$$\begin{aligned}\dot{X}_a &= -kX_a + 2\Omega X_{b_x} + \sqrt{2k}X_{a_{\mathrm{in}}} \\ \dot{Y}_a &= -kY_a + \sqrt{2k}Y_{a_{\mathrm{in}}} \\ \dot{X}_{b_x} &= -\Gamma X_{b_x} + X_{\zeta_x} \\ \dot{Y}_{b_x} &= -\Gamma Y_{b_x} - 2\Omega Y_a + Y_{\zeta_x}\end{aligned} \tag{6}$$

where $X_a = a + a^{\dagger}$, $X_{b_x} = \widetilde{b}_x + \widetilde{b}_x^{\dagger}$, $Y_a = -\mathrm{i}(a - a^{\dagger})$, $Y_{b_x} = -\mathrm{i}(\widetilde{b}_x - \widetilde{b}_x^{\dagger})$, and we have set $\phi_{L_1} = \phi_{L_2} = \pi/2$. The effective interaction Hamiltonian between the cavity and motional mode is simply a coupling of the form

$$H_{\mathrm{eff}} = \hbar\Omega Y_a X_{b_x}. \tag{7}$$

This is the interaction Hamiltonian of the cross-Kerr effect [19], which is the main result of this paper. The important feature of this Hamiltonian is that the amplitude quadrature X_a of cavity field picks up information about the amplitude quadrature of the motional mode X_{b_x}, while the latter is left unchanged.

3 Generation of squeezing and entanglement of motion

The Hamiltonian of equation (7) is identical to the one of the off-resonant interaction between the laser field and the atomic ensemble [14]. The spin squeezing and entanglement of two macroscopic atomic samples have been produced experimentally by the QND measurements with this Hamiltonian [15]. The protocols of quantum communication between atomic ensembles have also been proposed, *i.e.* quantum teleportation, quantum swapping. Thus, these protocols may be applied easily in motional state in cavity QED system. The QND schemes produce conditional squeezed motional states that are dependent on the measurement result. On the other hand, the unconditional squeezing may be generated by quantum feedback [20]. The results of the QND measurement, which conditionally squeeze the motion, are used to drive the system into the desired, deterministic, squeezed motional state.

First we discuss the scheme of producing the EPR state between two atoms in the same cavity. Two atoms are located at the different nodes of the cavity field. The amplitude quadrature of the cavity field output is measured. The equations of motion for the operators are

$$\begin{aligned}\dot{a} &= -ka + \mathrm{i}\Omega\left(\mathrm{e}^{-\mathrm{i}\phi_{L_1}^1}\widetilde{b}_{1x} + \mathrm{e}^{-\mathrm{i}\phi_{L_2}^1}\widetilde{b}_{1x}^{\dagger} + \mathrm{e}^{-\mathrm{i}\phi_{L_1}^2}\widetilde{b}_{2x}\right. \\ &\quad \left.+\mathrm{e}^{-\mathrm{i}\phi_{L_2}^2}\widetilde{b}_{2x}^{\dagger}\right) + \sqrt{2k}a_{\mathrm{in}} \\ \dot{\widetilde{b}}_{1x} &= -\Gamma\widetilde{b}_{1x} + \mathrm{i}\Omega\left(a^{\dagger}\mathrm{e}^{-\mathrm{i}\phi_{L_2}^1} + a\mathrm{e}^{\mathrm{i}\phi_{L_1}^1}\right) + \zeta_{1x} \\ \dot{\widetilde{b}}_{2x} &= -\Gamma\widetilde{b}_{2x} + \mathrm{i}\Omega\left(a^{\dagger}\mathrm{e}^{-\mathrm{i}\phi_{L_2}^2} + a\mathrm{e}^{\mathrm{i}\phi_{L_1}^2}\right) + \zeta_{2x}.\end{aligned} \tag{8}$$

Here, $\widetilde{b}_{1x}$ and $\widetilde{b}_{2x}$ are annihilation operators for the two quantized atomic motions respectively. $\phi_{L_1}^1$, $\phi_{L_2}^1$ and $\phi_{L_1}^2$, $\phi_{L_2}^2$ are the phase of two laser fields of atom 1 and 2, respectively. Firstly, we set $\phi_{L_1}^1 = \phi_{L_2}^1 = \phi_{L_1}^2 = \phi_{L_2}^2 = \pi/2$ at time t_1, then the quadratures of the cavity field and motional mode are

$$\begin{aligned}\dot{X}_a &= -kX_a + 2\Omega(X_{b_{1x}} + X_{b_{2x}}) + \sqrt{2k}X_{a_{\mathrm{in}}} \\ \frac{\mathrm{d}}{\mathrm{d}t}(Y_{b_{1x}} + Y_{b_{2x}}) &= -\Gamma(Y_{b_{1x}} + Y_{b_{2x}}) - 4\Omega Y_a + Y_{\zeta_{1x}} + Y_{\zeta_{2x}}\end{aligned} \tag{9}$$

and Y_a, $(X_{b_{1x}} + X_{b_{2x}})$, $(X_{b_{1x}} - X_{b_{2x}})$, $(Y_{b_{1x}} - Y_{b_{2x}})$ are not changed. In the realistic case $k \gg \Omega$, we can adiabatically eliminate the cavity mode a

$$X_a \simeq 2\frac{\Omega}{k}(X_{b_{1x}} + X_{b_{2x}}) + \sqrt{\frac{2}{k}}X_{a_{\mathrm{in}}}. \tag{10}$$

The experimentally measured quantity is the integration of the homodyne photon current over the measurement time T. With equation (10) and the boundary condition $a_{\mathrm{out}} = a - a_{\mathrm{in}}$, the measured observable corresponds to the operator [21]

$$\begin{aligned}X_T &= \frac{1}{T}\int_0^T \left(a_{\mathrm{out}}(t) + a_{\mathrm{out}}^{\dagger}(t)\right)\mathrm{d}t \\ &= \frac{2\sqrt{2}\Omega}{\sqrt{k}}(X_{b_{1x}} + X_{b_{2x}}) + \frac{1}{\sqrt{T}}X_T^{\mathrm{in}}\end{aligned} \tag{11}$$

where $X_T^{\mathrm{in}} = a_T + a_T^{\dagger}$, and a_T, satisfying $[a_T, a_T^{\dagger}] = 1$, is defined by $a_T = 1/\sqrt{T}\int_0^T a_{\mathrm{out}}(t)\mathrm{d}t$. Equation (11) assumes $k \gg \Omega$ and $\mathrm{e}^{-kT} \ll 1$. There are two different contributions in equation (11). The first term represents the signal, which is proportional to $X_{b_{1x}} + X_{b_{2x}}$, and the second term is the vacuum noise. We get a collective measurement of $X_{b_{1x}} + X_{b_{2x}}$ with some vacuum noise $X_{a_{\mathrm{in}}}$ from the cavity field output. After this round of measurements, we set $\phi_{L_1}^1 = \phi_{L_2}^2 = \pi$ and $\phi_{L_2}^1 = \phi_{L_1}^2 = 0$, the quadratures are

$$\dot{X}_a = -kX_a + 2\Omega(Y_{b_{1x}} - Y_{b_{2x}}) + \sqrt{2k}X_{a_{\rm in}}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(X_{b_{1x}} - X_{b_{2x}}) = -\Gamma(X_{b_{1x}} - X_{b_{2x}}) + 4\Omega Y_a + X_{\zeta_{1x}} - X_{\zeta_{2x}} \quad (12)$$

and Y_a, $(X_{b_{1x}} + X_{b_{2x}})$, $(Y_{b_{1x}} + Y_{b_{2x}})$, $(Y_{b_{1x}} - Y_{b_{2x}})$ keep unchanged. In this round, we get the collective measurement of new variables $Y_{b_{1x}} - Y_{b_{2x}}$. In this way, both the EPR operators $X_{b_{1x}} + X_{b_{2x}}$ and $Y_{b_{1x}} - Y_{b_{2x}}$ are measured, and the final state of the two atomic motion is collapsed into a two-mode squeezed state with variance

$$\left\langle \delta^2(X_{b_{1x}} + X_{b_{2x}})\right\rangle/2 = \left\langle \delta^2(Y_{b_{1x}} - Y_{b_{2x}})\right\rangle/2 = \mathrm{e}^{-2r} \quad (13)$$

where the squeezing parameter r is given by

$$r = \frac{1}{2}\ln\left(1 + 2\eta^2\right) \quad (14)$$

where $\eta = \sqrt{8\Omega^2 T/k}$ and the atomic motion decay during the measurements is not considered. Thus, using only coherent light, we generate continuous variable entanglement between two nonlocal atomic motions.

We now consider the two atoms placed in separate cavities, and interacting sequentially with the same light field. That is, the outgoing field from the first cavity enters the second one. We assume the same coupling constant, the same cavity field rate and the same oscillation frequency for the two atoms. Firstly, we set $\phi^1_{L_1} = \phi^1_{L_2} = \phi^2_{L_1} = \phi^2_{L_2} = \pi/2$, then the quadratures of the cavity field and motional mode of atom 1 are

$$a \simeq \frac{\Omega}{k}X_{b_{1x}} + \sqrt{\frac{2}{k}}a_{\rm in}$$

$$\dot{Y}_{b_{1x}} = -2\sqrt{\frac{2}{k}}\Omega Y_{a_{\rm in}} - \Gamma Y_{b_{1x}} + Y_{\zeta_{1x}} \quad (15)$$

where, we consider adiabatically setting $\dot{a} = 0$ when the cavity field rate $k \gg \Omega$. The boundary condition reads $a'_{\rm in} = a_{\rm out} = \sqrt{2k}a$. Then the quadratures of the output optical field from cavity 2 and motional mode of atom 2 are

$$X_{a'_{\rm out}} = 2\Omega\sqrt{\frac{2}{k}}(X_{b_{1x}} + X_{b_{2x}}) + X_{a_{\rm in}}$$

$$\dot{Y}_{b_{2x}} = -2\sqrt{\frac{2}{k}}\Omega Y_{a_{\rm in}} - \Gamma Y_{b_{2x}} + Y_{\zeta_{2x}}. \quad (16)$$

We measure the amplitude quadrature of the output optical field from cavity 2 to achieve the information of $X_{b_{1x}} + X_{b_{2x}}$ with the measurement time T. After this round of measurements, we set $\phi^1_{L_1} = \phi^2_{L_2} = \pi$ and $\phi^1_{L_2} = \phi^2_{L_1} = 0$, the quadratures are

$$X_{a'_{\rm out}} = 2\Omega\sqrt{\frac{2}{k}}(Y_{b_{1x}} - Y_{b_{2x}}) + X_{a_{\rm in}}$$

$$\dot{X}_{b_{1x}} = 2\sqrt{\frac{2}{k}}\Omega Y_{a_{\rm in}} - \Gamma X_{b_{1x}} + X_{\zeta_{1x}}$$

$$\dot{X}_{b_{2x}} = -2\sqrt{\frac{2}{k}}\Omega Y_{a_{\rm in}} - \Gamma X_{b_{2x}} + X_{\zeta_{2x}} \quad (17)$$

and $Y_{a_{\rm in}}$, $Y_{b_{1x}}$, $Y_{b_{2x}}$ are not changed. In this round, we get the collective measurement of new variables $Y_{b_{1x}} - Y_{b_{2x}}$. In this way, both the EPR operators $X_{b_{1x}} + X_{b_{2x}}$ and $Y_{b_{1x}} - Y_{b_{2x}}$ are measured from the output optical field of cavity 2, and the final state of the two atomic motion in separate cavities is collapsed into a two-mode squeezed state with variance $\langle\delta^2(X_{b_{1x}} + X_{b_{2x}})\rangle/2 = \langle\delta^2(Y_{b_{1x}} - Y_{b_{2x}})\rangle/2 = \mathrm{e}^{-2r}$, where the squeezing parameter r is equal to equation (14).

In the following, we show as an example how to achieve quantum communication, *i.e.*, quantum teleportation, between distant atomic motion using only coherent light. We consider unconditional quantum teleportation of continuous variables [1] from one atomic motion to the other in separate cavities since we have continuous variable entanglement. To achieve quantum teleportation, first two distant atomic motion 1 and 2 are prepared in a continuously entangled state using the nonlocal Bell measurement described above. Then, a Bell measurement with the same setup on the two local atom 1 and 3, together with a straightforward displacement of $X_{b_{2x}}$, $Y_{b_{2x}}$ on the atom 2, will teleport an unknown atomic motion state from atom 3 to 2. The teleportation quality is best described by the fidelity, which, for a pure input state, is defined as the overlap of the teleported state and the input state. For any coherent input state of the atom 3, the teleportation fidelity is given by

$$F = 1/\left(1 + \frac{1}{1+2\eta^2} + \frac{1}{2\eta^2}\right). \quad (18)$$

We now consider briefly the conditions under which the most significant assumptions required by our model should be satisfied. These conditions has been examined in some detail in reference [9]. First, the neglect of terms in the effective motion-cavity mode interaction Hamiltonians, which vary like $\mathrm{e}^{\pm 2\mathrm{i}v_x t}$ requires that the trap frequencies be large in comparison with the cavity decay rate k and the effective coupling parameter Ω. Second, the Lamb-Dicke parameter must satisfy $\eta_x \ll 1$. Let us consider a specific example of the trapped $^9Be^+$ ions. Recent experiments with this ion [7] have been performed with harmonic oscillation frequency along the principal axis of the trap $v_x/2\pi \simeq 11$–30 MHz, corresponding to Lamb-Dicke parameter $\eta_x \simeq 0.14$–0.086. If we assume, for example, that the mirrors forming the cavity have radii of curvature equal to 5 cm and are separated by a distance $l = 1$ mm, the $g_0/(2\pi) = 5.3$ MHz. For a cavity finesse of 75 000 one obtains $k/(2\pi) = 1$ MHz, while a trap frequency of $v_x/(2\pi) = 22$ MHz (corresponding to a Lamb-Dicke parameter $\eta_x = 0.1$) gives $v_x/k = 22$. We choose the measuring time $T \sim 8$ μs and let Kerr coefficient $\Omega/(2\pi) = 0.1$ MHz. A high squeezing $r = 1.1$ and a fidelity for the teleportation $F = 0.81$ is obtained if there is no extra noise.

4 Conclusion

In conclusion, we have described a scheme for squeezing and entangling motional modes *via* QND measurements in cavity QED. We have considered two scenarios to entangle the motions of two atoms: atoms in same and in separate cavities. This scheme has a advantage of establishing entanglement and achieving some protocols for quantum communication between atomic motions only by means of coherent optical field in cavity QED. The exciting progress has been made recently in experimental cavity QED with single trapped atoms [22], which collectively offers great encouragement to our proposal. Realization of this scheme would offer the exciting possibility of implementing a variety of continuous variable quantum computation and communication protocols.

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